CHERN-SIMONS-MATTER THEORIES AT STRONG COUPLING

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Based on [M.M.-Putrov, 0912.1458] [Drukker-M.M.-Putrov, 1007.1453] [Couso Santamaria-M.M.-Putrov, 1011.6281] Two well-known virtues of large N string/gauge theory dualities:

• The large radius limit of string theory is dual to the strong coupling regime in the gauge theory

$$\frac{R}{\ell_s} \gg 1 \leftrightarrow \lambda \gg 1$$

• The genus expansion of the string theory can be in principle mapped to the *I/N* expansion of the gauge theory

These virtues have their counterparts:

 It is hard to test the duality, since one has to do calculations at strong 't Hooft coupling in the gauge theory. More ambitiously, one would like to have results interpolating between weak and strong coupling

• It is hard to obtain information beyond the planar limit, even in the gauge theory side.

In this talk I will report on some recent progress on these problems in Chern-Simons-matter theories and their string duals.

In particular, I will present exact results (interpolating functions) for the planar free energy of these theories on the thee-sphere.

The strong coupling limit is in perfect agreement with the AdS predictions , and in particular provides the first quantitative test of the $N^{3/2}$ behaviour of the M2 brane theory

Moreover, I will show that the strong coupling limit of the theory admits a simple geometric description in terms of *tropical geometry* The analysis of the ABJM theory will rely on the following "chain of dualities", which relates it a topological gauge/string theory via a matrix model:





A B J M theory



2 twisted hypers

 $\begin{array}{ll} \text{two `t Hooft} & \\ \text{couplings} & \lambda_i = \frac{N_i}{k} \end{array}$

$$U(N_1)_k \times U(N_2)_{-k}$$

CS theories + 4 hypers C in the bifundamental; related to supergroup $U(N_1|N_2)$ theory via [Gaiotto-Witten]

In this talk we restrict to the "ABJM slice" $\lambda_1 = \lambda_2 = \lambda = \frac{N}{k}$

This is a 3d SCFT which (conjecturally) describes N M2 branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity

Gravity dual

M-theory on AdS₄ × $\mathbb{S}^7/\mathbb{Z}_k$



Gauge/gravity dictionary:

$$\left(\frac{L}{\ell_s}\right)^2 = \left(32\pi^2\lambda\right)^{1/2}$$

$$g_{\rm st} = \frac{1}{k} \left(32\pi^2 \lambda \right)^{1/4}$$

A string/gravity prediction

The planar free energy of the Euclidean theory on S^3 should be given by the (regularized) Euclidean Einstein-Hilbert action on AdS4

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \sinh^2(\rho) \,\mathrm{d}\Omega^2_{\mathbb{S}^3},$$



Recent arguments vindicating this quantity as a measure of the number of degrees of freedom [Jafferis]

Localization and matrix models

In supersymmetric theories one can often reduce the path integral to an integral over supersymmetric configurations/ vacua. On spherical spacetimes all the modes are massive and the vacua simplify dramatically: the path integral reduces to a matrix model

Main example: in N=4 SYM the path integral calculating the vev of the I/2 BPS Wilson loop reduces to a *Gaussian, Hermitian matrix model* [Ericksson-Semenoff-Zarembo, Drukker-Gross, Pestun]

$$\langle W_R \rangle = \frac{1}{Z} \int dM \, \mathrm{e}^{-\frac{2N}{\lambda} \mathrm{Tr} \, M^2} \mathrm{Tr}_R \mathrm{e}^M \qquad \lambda = g_{\mathrm{YM}}^2 N$$

This is the simplest matrix model, and the planar density of eigenvalues is the famous Wigner semicircle distribution



From this we can obtain an exact interpolating function for the Wilson loop vev:

$$\frac{1}{N} \langle W_{\Box} \rangle_{\text{planar}} = \int_{-\sqrt{\lambda}}^{\sqrt{\lambda}} \rho(z) e^{z} dz = \frac{2}{\sqrt{\lambda}} I_{1}(\sqrt{\lambda})$$

Reduction to a matrix model in ABJM

Localization techniques were applied to the ABJM theory in a beautiful paper by [Kapustin-Willett-Yaakov]. The partition function on \mathbb{S}^3 is given by the following matrix integral:



We "just" need the planar solution, but exact in the 't Hooft parameters, in order to go to strong coupling

Relation to Chern-Simons matrix models

Shortcut: relate this to the lens space CS matrix model [M.M. building on Lawrence-Rozansky] [AKMV, Halmagyi-Yasnov]

U(N) (pure) CS theory on \mathbb{S}^3 :

$$Z_{\mathbb{S}^3}(N, g_{\text{top}}) = \frac{1}{N!} \int \prod_{i=1}^N \frac{\mathrm{d}\mu_i}{2\pi} \prod_{i < j} \left(2\sinh\left(\frac{\mu_i - \mu_j}{2}\right) \right)^2 \mathrm{e}^{-\frac{1}{2g_{\text{top}}}\sum_i \mu_i^2}$$

can be rederived with SUSY localization [Kapustin et al.]

U(N) (pure)
CS theory on
L(2,I)=
$$S^3/\mathbb{Z}_2$$

$$Z_{L(2,1)}(N, g_{top}) = \sum_{N_1+N_2=N} Z_{L(2,1)}(N_1, N_2, g_{top})$$

$$I_{L(2,I)} = S^3/\mathbb{Z}_2$$

$$I_{L(2,1)}(N, g_{top}) = \sum_{N_1+N_2=N} Z_{L(2,1)}(N_1, N_2, g_{top})$$
sum over flat connections

$$Z_{L(2,1)}(N_1, N_2, g_{\text{top}}) = \frac{1}{N_1! N_2!} \int \prod_{i=1}^{N_1} \frac{\mathrm{d}\mu_i}{2\pi} \prod_{j=1}^{N_2} \frac{\mathrm{d}\nu_j}{2\pi} \prod_{i$$

This is a *two-cut* model with two 't Hooft parameters $t_i = g_{top}N_i$



Superficially similar to the matrix model describing ABJM...

I/N expansion
$$F(N_1, N_2, g_{top}) = \sum_{g \ge 0} g_{top}^{2g-2} F_g(t_1, t_2)$$

Fact: The ABJM MM is the supermatrix version of the L(2, I) MM. They are related by $t_2 \rightarrow -t_2$ [M.M.-Putrov]

i.e.
$$t_1 = 2\pi i \lambda_1, \quad t_2 = -2\pi i \lambda_2$$

CS matrix model ABJM theory

The I/N expansion of the lens space matrix model gives the I/N expansion of the ABJM free energy on the three-sphere

Planar solution: matrix model approach

The planar solution of the CS lens space matrix model has been known for some time [AKMV, Halmagyi-Yasnov]. The solution is elegantly encoded in a *resolvent* or *spectral curve*

$$\omega_0(z) = 2 \log \left(\frac{e^{-t/2}}{2} \left[\sqrt{(Z+b)(Z+1/b)} - \sqrt{(Z-a)(Z-1/a)} \right] \right)$$

$$t = t_1 + t_2$$

$$C_1$$

$$C_2$$

$$C_2$$

$$C_$$

 $\rho_k(z) = -\frac{t}{t_k} \frac{1}{2\pi i} \left(\omega_0(z + i\epsilon) - \omega_0(z - i\epsilon) \right)$

the cuts=densities

All the planar information is given by period integrals of the resolvent

$$t_i = \frac{1}{4\pi i} \oint_{\mathcal{C}_i} \omega_0(z) dz, \quad i = 1, 2$$

$$\frac{\partial F_0}{\partial t_1} - \frac{\partial F_0}{\partial t_2} = -\frac{1}{2} \oint_{\mathcal{D}} \omega_0(z) \mathrm{d}z$$

We have to understand what are the weak and the strong coupling limits in terms of the geometry of the curve. In ABJM theory we also want the 't Hooft parameters to be *imaginary* It turns out that all quantities appearing here are naturally expressed in terms of a real variable κ , closely related to the positions of the cuts

weak coupling	strong coupling
$\kappa = 0$	$\kappa = \infty$
$a, b \sim 1$	$a \sim i\kappa, b \sim -i\kappa$
$\lambda \sim \kappa$	$\lambda \sim \log^2(\kappa)$
$\partial_{\lambda} F_0 \sim \kappa \log \kappa$	$\partial_{\lambda} F_0 \sim \log \kappa$

$$F_0(\lambda) \sim \lambda^{3/2}, \quad \lambda \gg 1$$

We can in fact write very explicit interpolating functions:

$$\lambda(\kappa) = \frac{\kappa}{8\pi} {}_{3}F_{2}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -\frac{\kappa^{2}}{16}\right)$$
$$\partial_{\lambda}F_{0}^{\text{orb}}(\lambda) = \frac{\kappa}{4}G_{3,3}^{2,3}\left(\begin{array}{cc}\frac{1}{2}, & \frac{1}{2}, & \frac{1}{2}\\ 0, & 0, & -\frac{1}{2}\end{array}\middle| -\frac{\kappa^{2}}{16}\right) + 4\pi^{3}i\lambda$$



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Geometry of the spectral curve

Since Z is an exponentiated variable, it is given geometrically by a cylinder. The curve is a double covering branched over two cuts.



After setting $\log Y = \omega_0(Z)$ the equation for the curve reads

$$Y + \frac{Z^2}{Y} - Z^2 + i\kappa Z - 1 = 0 \quad \text{in } \mathbb{C}^* \times \mathbb{C}^*$$

The strong coupling limit $\kappa \to \infty$ is known as the *tropical limit* of the curve

In this limit the cylinder shrinks to a line, and the curve can be represented as a set of segments where the relation between log Z and log Y is linear



The calculation of the periods in this limit becomes elementary and one gets the AdS results for the free energy on the nose The density of eigenvalues, given by the discontinuity across the cuts, is *constant* in the tropical/strong coupling limit. This can be also shown in a direct analysis of the matrix model [Herzog-Klebanov-Pufu-Tesileanu]



Adding matter

We can deform ABJM by adding Nf matter fields in the (anti)fundamental. The resulting theory has N=3 SUSY and its M-theory dual involves a tri-Sasakian manifold

$$\begin{split} F_{\mathcal{N}=3}(\mathbb{S}^3) &= -N^2 \frac{2\pi}{3\sqrt{2\lambda}} \frac{1 + N_f/k}{\sqrt{1 + N_f/(2k)}} \\ \text{quenched} \\ \text{approximation} &= -N^2 \frac{2\pi}{3\sqrt{2\lambda}} - \frac{\pi}{4} N_f N \sqrt{2\lambda} + \mathcal{O}(N_f^2) \\ N_f \ll N \end{split}$$

The free energy of this theory can be also calculated by a matrix model, which is a perturbation of the ABJM matrix model with determinant-like operators

$$\exp\left[-N_f^{(1)}\sum_{j=1}^N \log\left(2\cosh\frac{\mu_j}{2}\right) - N_f^{(2)}\sum_{j=1}^N \log\left(2\cosh\frac{\nu_j}{2}\right)\right]$$
$$N_f = N_f^{(1)} + N_f^{(2)}$$

In the quenched approximation this insertion can be evaluated perturbatively in the Nf's, and it reduces to the calculation of vevs in the ABJM matrix model It is possible to solve the matrix model in the Veneziano limit $N_f \sim N \gg 1$ (i.e. unquenched flavor). One can use again tropical techniques to compute the free energy at strong coupling and reproduce the AdS results.



Other results/perspectives in this approach

I) Exact interpolating functions for Wilson loops

- 2) Exact results at all orders in the genus expansion (as in noncritical strings!) and nonperturbative effects in the string coupling constant [to appear]
 - 3) Exact results for more complicated quiver theories [Herzog et al.]
 - 4) Connections to topological string theory and toric/ tropical geometry

Conclusion: a very powerful method to obtain exact results in Chern-Simons-matter theories and to test AdS dualities. The strong coupling expansion can be implemented directly as an expansion around the tropical limit.